

Vector Meson Mixing and Charge Symmetry Violation**[Phys. Lett. B370 (1996) 12-16]**H.B. O'Connell^a, A.G. Williams^a, M. Bracco^b and G. Krein^b*^aDepartment of Physics and Mathematical Physics, University of Adelaide,
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Abstract

We discuss the consistency of the traditional vector meson dominance (VMD) model for photons coupling to matter, with the vanishing of vector meson-meson and meson-photon mixing self-energies at $q^2 = 0$. This vanishing of vector mixing has been demonstrated in the context of rho-omega mixing for a large class of effective theories. As a further constraint on such models, we here apply them to a study of photon-meson mixing and VMD. As an example we compare the predicted momentum dependence of one such model with a momentum-dependent version of VMD discussed by Sakurai in the 1960's. We find that it produces a result which is consistent with the traditional VMD phenomenology. We conclude that comparison with VMD phenomenology can provide a useful constraint on such models.

The experimental extraction of $\Pi_{\rho\omega}$ (in the pion EM form-factor [1]) is in the timelike q^2 region around the ρ – ω mass, yet it is used to generate charge symmetry violation (CSV) in boson exchange models of the NN interaction in the spacelike region [2,3]. The traditional assumption was that the mixing amplitude was independent of q^2 .

This assumption was first questioned by Goldman *et al.* [4] who constructed a model in which the ρ and ω mixed via a quark loop contribution which is non-vanishing if and only if $m_u \neq m_d$. Their conclusion of a significant momentum dependence was subsequently supported by other studies, which included an analogous NN -loop calculation [5] using the n - p mass difference and more elaborate quark-loop model calculations [6]. All of these predicted a similar momentum-dependence for $\Pi_{\rho\omega}(q^2)$ with a node near the origin ($q^2 = 0$). At a more formal level, it was subsequently shown that the vector-vector mixings must identically vanish at $q^2 = 0$ in a large class of effective theories [7] where the mixing occurs exclusively through coupling of the vector mesons to conserved currents and where the vector currents commute in the usual way. Recent work in chiral perturbation theory and QCD sum rules has also suggested that such mixing matrix elements must, in general, be expected to be momentum dependent [8].

In response to this, alternative mechanisms involving CSV have been proposed [9]. Indeed, as the vector mesons are off shell, the individual mechanisms should not be examined in isolation, because they are dependent on the choice of interpolating fields for the vector mesons and are not physical quantities. It has been argued that one could find a set of interpolating fields for the rho and omega such that *all* nuclear CSV occurs through a constant ρ – ω mixing with the CSV vertex contributions vanishing [10]. However this possibility has been questioned on the grounds of unitarity and analyticity [11].

The same models which have been used to examine the question of ρ – ω mixing can also be applied to studies of ρ – γ mixing. They can then be compared to phenomenology and vector meson dominance (VMD) models, which have *traditionally* assumed the coupling of the photon to the rho was independent of q^2 . The first person to raise this question was Miller [12]. The purpose of this letter is to carefully explore the issues raised and compare numerical predictions for such a mixing model with experimental data. As discussed recently [13], the appropriate representation of VMD to use with a momentum dependent photon-rho coupling is VMD1, given by the Lagrangian [14]

$$\mathcal{L}_1 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu - g_{\rho\pi\pi}\rho_\mu J^\mu - eA_\mu J^\mu - \frac{e}{2g_\rho}F_{\mu\nu}\rho^{\mu\nu} + \dots \quad (1)$$

where J_μ is the hadronic current and $F_{\mu\nu}$ and $\rho_{\mu\nu}$ are the EM and ρ field strength tensors respectively (the dots refer to the hadronic part of the Lagrangian). From this we obtain the VMD1 expression for the form-factor for the pion [13]

$$F_\pi^{(1)}(q^2) = 1 - \frac{q^2 g_{\rho\pi\pi}}{g_\rho[q^2 - m_\rho^2 + im_\rho\Gamma_\rho]}. \quad (2)$$

Note that in the limit of exact universality $g_{\rho\pi\pi} = g_\rho$ and we recover the usual VMD2 model prediction for the pion form-factor [13,14]. Recall that in this traditional VMD

(i.e., VMD2) model the photon couples to hadrons *only* through first coupling to vector mesons with a *constant* coupling strength, e.g., for the ρ – γ coupling we have $\Pi_{\rho\gamma}^{\text{VMD2}}(q^2) \equiv -m_\rho^2 e/g_\rho$

We shall define a VMD1-like model to be one in which the photon couples to the hadronic field both directly and via a q^2 -dependent coupling (with a node at $q^2 = 0$) to vector mesons. A VMD1-like model may differ from pure VMD1 as the coupling of the photon to the rho (generated by some microscopic process) will not generally be linear in q^2 . Hence g_ρ , which is a constant in VMD1 (and VMD2 as they share the same g_ρ [13, 14]), may acquire some momentum dependence in a VMD1-like model; the test for the phenomenological validity of the model is then that this momentum dependence for g_ρ is not too strong. For example, we can easily determine the coupling of the photon to the pion field via the rho meson for a VMD1-like model. We note the appearance in Eq. (3) of the photon-rho mixing term, $\Pi_{\rho\gamma}^{\mu\nu}(q^2)$, which can be determined from Feynman rules, and which will, in general, be q^2 -dependent. Such an analysis gives for any VMD1-like model

$$\begin{aligned} -i\mathcal{M}^\mu(q^2) &\equiv -ie(p^+ - p^-)_\sigma [D_\gamma(q^2)]^{\mu\sigma} F_\pi(q^2) \\ &= i[D_\gamma(q^2)]^{\mu\sigma} i[\Gamma_{\gamma\pi}(q^2)]_\sigma + i[D_\gamma(q^2)]^{\mu\sigma} i[\Pi_{\gamma\rho}(q^2)]_{\sigma\tau} i[D_\rho(q^2)]^{\tau\nu} i[\Gamma_{\rho\pi}(q^2)]_\nu \\ &= -ie(p^+ - p^-)_\sigma i[D_\gamma(q^2)]^{\mu\sigma} \left[1 + \frac{\Pi_{\rho\gamma}(q^2)}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho} \frac{g_{\rho\pi\pi}}{e} \right], \end{aligned} \quad (3)$$

where D , Π and Γ denote propagators, one-particle irreducible mixing amplitudes and proper vertices respectively. Here p^+ and p^- are the outgoing momenta of the π^+ and π^- respectively. For this model to reproduce the phenomenologically successful VMD, and hence provide a good fit to the data (assuming exact universality), $\Pi_{\rho\gamma}(q^2)$ and g_ρ must be related by (comparing Eqs. (2) and (3))

$$\Pi_{\rho\gamma}^{\text{VMD1}}(q^2) = -\frac{q^2 e}{g_\rho(q^2)}. \quad (4)$$

Note that this result then implies that $\Pi_{\rho\gamma}^{\text{VMD1}}(q^2) = (q^2/m_\rho^2)\Pi_{\rho\gamma}^{\text{VMD2}}(q^2)$. Eq. (4) arises from the simple VMD1 picture when universality is assumed and is also consistent with the usual VMD2 picture as explained elsewhere [13, 14].

Thus Eq. (4) is the central equation of this work, since vector-meson mixing models (e.g., ρ – ω mixing) can also be used to calculate ρ – γ mixing and then confronted with traditional VMD phenomenology. The results quoted in the review by Bauer et al. [15] are summarised in Tables I and XXXII of that reference. They list a range of values which vary depending on the details of the fit to the ρ mass (m_ρ) and width (Γ_ρ). Within the context of the traditional VMD (i.e., VMD2) framework they extract $g_\rho^2(q^2 = 0)/4\pi$ from ρ^0 photoproduction ($\gamma p \rightarrow \rho^0 p$) and $g_\rho^2(q^2 = m_\rho^2)/4\pi$ from $\rho^0 \rightarrow e^+e^-$. The three sets of results quoted are (in an obvious shorthand notation): $\Gamma_\rho = 135, 145, 155\text{MeV}$, $m_\rho = 767, 774, 776\text{MeV}$, $g_\rho^2(q^2 = 0)/4\pi = 2.43 \pm 0.10, 2.27 \pm 0.23, 2.18 \pm 0.22$, $g_\rho^2(q^2 = m_\rho^2)/4\pi = 2.21 \pm 0.017, 2.20 \pm 0.06, 2.11 \pm 0.06$ respectively. We see that g_ρ is a free parameter of the traditional VMD model (VMD2) which is adjusted to fit the available

cross section data. The central feature of the VMD2 model is that it presumes a constant value for its coupling constant g_ρ . We note in passing that the universality condition is $g_\rho \sim g_{\rho\pi\pi} \sim g_{\rho NN}^{\text{univ}} \sim g_{\rho\rho\rho}$ and where experimentally we find [15, 16] for each of these $g^2/4\pi \sim 2$. For example, the values of $g_{\rho\pi\pi}$ corresponding to the above three sets of results are $g_{\rho\pi\pi}^2(q^2 = m_\rho^2)/4\pi = 2.61, 2.77, 2.95$ and are extracted from $\rho^0 \rightarrow \pi^+\pi^-$. It should be noted that the ρNN interaction Lagrangian is here defined as in Refs. [3, 5] with no factor of two [14, 16] and hence $g_{\rho NN} = g_{\rho NN}^{\text{univ}}/2$. As a typically used value is $g_{\rho NN}^2/(4\pi) = 0.41$ we see that universality is not accurate to better than 40% in g_ρ^2 , which corresponds to $\simeq 20\%$ in g_ρ .

The results of the VMD2 analysis [15] are approximately consistent with g_ρ being a constant and so we see from Eq. (4) that $\Pi_{\rho\gamma}$ in VMD1-like models should not deviate too strongly from behaviour linear with q^2 .

We shall now examine the process within the context of the model used by Piekarawicz and Williams (PW) who considered ρ – ω mixing as being generated by a nucleon loop [5] within the Walecka model. Using nucleon loops as the intermediate states removes the formation of unphysical thresholds in the low q^2 region and allows us to use well-known parameters. The rho-coupling is not a simple, vector coupling, but rather [17]

$$\Gamma_{\rho NN}^\mu = g_{\rho NN}\gamma^\mu + i\frac{f_{\rho NN}}{2M}\sigma_{\mu\nu}q^\nu, \quad (5)$$

where $C_\rho \equiv f_{\rho NN}/g_{\rho NN} = 6.1$ and M is the nucleon mass. With the introduction of tensor coupling the model is no longer renormalisable, but to one loop order we can introduce some appropriate renormalisation prescription. As the mixings are transverse, we write $\Pi_{\mu\nu}(q^2) = (g_{\mu\nu} - q_\mu q_\nu/q^2)\Pi(q^2)$ [7]. The photon couples to charge, like a vector and so, unlike the PW calculation, we have only a proton loop to consider. Here we can safely neglect the coupling of the photon to the nucleon magnetic moment and so there is no neutron loop contribution nor any tensor-tensor contribution to the proton loop. This sets up two kinds of mixing, vector-vector $\Pi_{\text{vv}}^{\mu\nu}$ and vector-tensor $\Pi_{\text{vt}}^{\mu\nu}$, where (using dimensional regularisation with the associated scale, μ)

$$\Pi_{\text{vv}}(q^2) = -q^2 \frac{eg_{\rho NN}}{2\pi^2} \left[\frac{1}{3\epsilon} - \frac{\gamma}{6} - \int_0^1 dx \, x(1-x) \ln \left(\frac{M^2 - x(1-x)q^2}{\mu^2} \right) \right], \quad (6)$$

$$\Pi_{\text{vt}}(q^2) = -q^2 \frac{eg_{\rho NN}}{8\pi^2} \left[\frac{1}{\epsilon} - \gamma - \int_0^1 dx \, \ln \left(\frac{M^2 - x(1-x)q^2}{\mu^2} \right) \right]. \quad (7)$$

Note that these functions vanish at $q^2 = 0$, as expected from the node theorem since we have coupling to conserved currents [7]. To remove the divergence and scale-dependence we add a counter-term

$$\mathcal{L}_{CT} = e \frac{g_{\rho NN} C_T}{2\pi^2} \rho_{\mu\nu} F^{\mu\nu}$$

to the Lagrangian in a minimal way so as to renormalise the model to one loop. This will contribute $-iC_T g_{\rho NN} e q^2/\pi^2$ to the photon-rho vertex, which will add to the contribution $i\Pi$ generated by the nucleon loop. The counter-term will contain pieces proportional to

$1/\epsilon$, γ and $\ln \mu^2$ to cancel the similar terms in Eqs. (6) and (7), and a constant piece, β , which will be chosen to fit the extracted value for $g_\rho(0)$. The counter-term is

$$C_T = -\frac{1}{\epsilon} \left(\frac{1}{6} + \frac{C_\rho}{8} \right) + \gamma \left(\frac{1}{12} + \frac{C_\rho}{8} \right) - \left(\frac{1}{12} + \frac{C_\rho}{8} \right) \ln \mu^2 + \beta, \quad (8)$$

which gives us the renormalised mixing,

$$\begin{aligned} \Pi_{\rho\gamma}(q^2) &= q^2 \frac{e g_{\rho NN}}{\pi^2} \left[\frac{1}{2} \left(\frac{5}{18} + \frac{2M^2}{3q^2} - \frac{8M^4 + 2M^2 q^2 - q^4}{3q^3 \sqrt{4M^2 - q^2}} \arctan \sqrt{\frac{q^2}{4M^2 - q^2}} \right. \right. \\ &\quad \left. \left. - \frac{\ln M^2}{6} \right) + \frac{C_\rho}{8} \left(-2 + 2 \sqrt{\frac{4M^2 - q^2}{q^2}} \arctan \sqrt{\frac{q^2}{4M^2 - q^2}} + \ln M^2 \right) - \beta \right]. \end{aligned} \quad (9)$$

We find that the choice $\beta = 8.32$ in our counter-term approximately reproduces the extracted value of $g_\rho(0)$ at $q^2 = 0$.

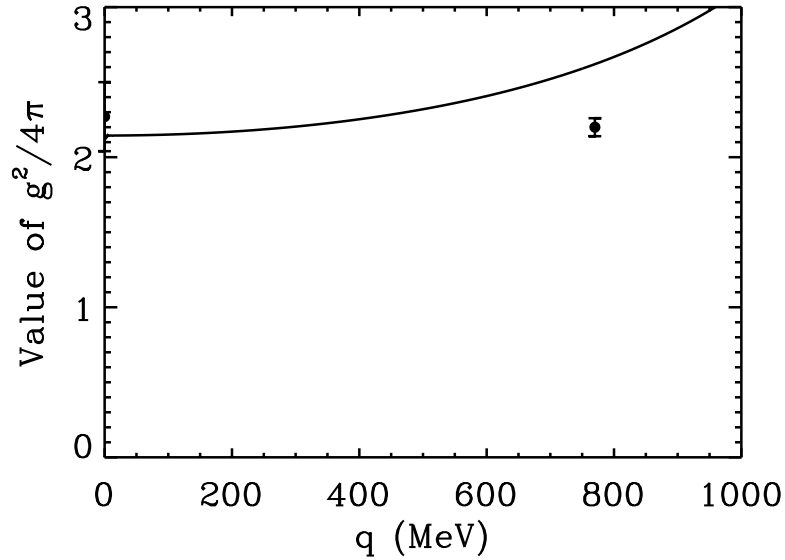


Figure 1: The PW model prediction for the mixing amplitude is related to the traditional VMD coupling $g_\rho(q^2)$ using the central result of Eq. (4). The resulting behaviour of $g_\rho^2/4\pi$ versus $q \equiv \sqrt{q^2}$ is then plotted in the timelike region for this model. Shown for comparison are a typical pair of results (2.27 ± 0.23 at $q = 0$ and 2.20 ± 0.06 at $q = m_\rho$, see text) taken from a traditional VMD based analysis of cross section data in Ref. [15].

The results for $g_\rho(q^2)$ for the PW model are shown in Fig. 1. We see that despite this model having a node in the photon-rho mixing at $q^2 = 0$ the resulting q^2 dependence of g_ρ is small. As can be seen from this plot, we obtain values of $g_\rho^2(0)/(4\pi) = 2.14$ and $g_\rho^2(m_\rho^2)/(4\pi) = 2.6$ compared to the experimental averages 2.3 and 2.17 respectively.

It should be remembered that Eq. (4) is only as reliable as universality, which is itself violated at a level of 30-40% . Based on this important observation, we can conclude then that the PW model provides a result consistent with the spread of extracted results given in Ref. [15]. It should be noted that any VMD1-like model which predicts a significantly greater deviation from linearity with q^2 will fail to reproduce phenomenology because of Eq. (4).

In summary, we have explicitly shown in Eq. (4) that the vanishing of vector-vector mixing at $q^2 = 0$ is completely consistent with the standard phenomenology of vector meson dominance (VMD). We have, in addition, applied the same type of model used in a study of $\rho - \omega$ mixing to extract the momentum dependence of $\rho - \gamma$ mixing and have compared the result to the VMD2 based analysis of the experimental data. We see that the phenomenological constraints of VMD can provide a useful independent test of VMD-like models of vector mixing and future studies should take adequate account of this. It would, of course, be preferable to reanalyse the data used in Ref. [15] from the outset using VMD1 rather than VMD2, but this more difficult task is left for future investigation.

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